B.Tech. SEM -IV (Chemical) 2014 Course (CBCS) : WINTER - 2018
SUBJECT: ENGINEERING MATHEMATICS-III

Day: Tuesday **W-2018-2326** Time: 02.30 PM TO 05.30 PM
Date: 13/11/2018 Max. Marks: 60

N.B:

1) All questions are **COMPULSORY.**
2) Figures to the right indicate **FULL** marks.
3) Use non programmable **CALCULATOR** allowed.
4) Assume suitable data if necessary.

Q.1 **a)** Solve: $\left(D^3+D\right)y=\cos x$. **(05)**

b) Solve: $\frac{dx}{y}=\frac{dy}{-x}=\frac{dz}{2x-3y}$. **(05)**

OR

a) Solve: $\left(D^2+1\right)y=\cot x$ By the method of variation of parameters. **(05)**

b) Solve: $x^2\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+6y=x^5$. **(05)**

Q.2 A tightly stretched string of length l with fixed ends is initially in equilibrium position. It is set vibrating by giving each point a velocity $v_0\sin^3\frac{\pi x}{l}$. Find the displacement $y(x,t)$. **(10)**

OR

A rectangular plate is bounded by x = 0, x = a, y = 0, y = b. Its surfaces are insulated and temperature along three edges x = 0, x = a, y = 0 is maintained at 0^0C. While the fourth edge y = b is maintained at constant temperature v_0, until steady state is reached. Find u (x,y). (Use Heat equation $\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}=0$.) **(10)**

Q.3 **a)** Represent the following function in the Fourier integral form: **(05)**

$$f(x)=\begin{cases}\frac{\pi}{4}\sin x, & |x|\le\pi\\ 0, & |x|>\pi\end{cases}$$

b) Find the Fourier cosine transform of $f(x)=e^{-2x}+4e^{-3x}$. **(05)**

OR

Solve the integral equation. **(10)**

$$\int_0^\infty f(x)\cos\lambda x\,dx=\begin{cases}1-\lambda, & 0\le\lambda\le1\\ 0, & \lambda\ge1\end{cases}$$

And hence show that $\int_0^\infty\frac{\sin^2 t}{t^2}dt=\frac{\pi}{2}$.

P.T.O.

Q.4

a) Obtain the Laplace transform of the function $\frac{d}{dt}\left(\frac{\sin t}{t}\right)$. **(05)**

b) Find the inverse Laplace transform of $\frac{s+2}{s^2+4s+13}$. **(05)**

OR

a) Solve the following differential equation by using Laplace transform. **(05)**
$y''+y=0, y(0)=1, y'(0)=2.$

b) Obtain the Laplace transform of the function. **(05)**

$$F(t)=\begin{cases} 5\sin 3\left(t-\frac{\pi}{4}\right), & t>\frac{\pi}{4} \\ 0, & t<\frac{\pi}{4} \end{cases}$$

Q.5

a) If $\bar{r}.\frac{d\bar{r}}{dt}=0$, then show that $\bar{r}$ has constant magnitude. **(05)**

b) Find directional derivative of $\phi=xy^2+yz^3$ at point $(2,-1,1)$ in the direction of vector $\vec{i}+2\vec{j}+2\vec{k}$. **(05)**

OR

a) Show that $\overline{F}=\left(ye^{xy}\cos z\right)\vec{i}+\left(xe^{xy}\cos z\right)\vec{j}-\left(e^{xy}\sin z\right)\vec{k}$ is irrotational. Find ϕ *if* $\overline{F}=\nabla\phi$. **(05)**

b) Show that $\nabla^4\left(r^2\log r\right)=\frac{6}{r^2}$. **(05)**

Q.6

Verify Green's theorem in the plane for $\oint_c\left(xy+y^2\right)dx+x^2dy$ where C is the closed curve of the region bounded by $y=x$ *and* $y=x^2$. **(10)**

OR

Verify the divergence theorem for the function $\overline{F}=x\vec{i}+y\vec{j}+z^2\vec{x}$ over the cylindrical region bounded by $x^2+y^2=4, z=0, z=2$. **(10)**

* * * * *