B. Tech. SEM -I (Computer Science & Business Systems) (CBCS 2018 Course)
: WINTER - 2018

SUBJECT : MATHEMATICS – I

Day : Thursday
Date : 22/11/2018

Time : 10.00 AM To 01.00 PM
Max. Marks : 60

W-2018-2254

N. B. :

1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.
3) Use of non-programmable calculator is **ALLOWED**.

Q. 1 Solve:

a) $\int_0^{\infty}\int_0^{x} x e^{\frac{-x^2}{y}}\, dxdy$ **(05)**

b) $\int_0^{1}\int_{4y}^{4} e^{x^2}\, dxdy$ **(05)**

OR

a) Evaluate $\iint y\, dxdy$ over the area bounded by $y = x^2$ and $x + y = 2$. **(05)**

b) Evaluate $\iint_R \sqrt{xy\,(1 - x - y)}\, dxdy$, where R is the area bounded by **(05)**
$x = 0$, $y = 0$ and $x + y = 1$.

Q. 2 **a)** Find the volume cut from the sphere **(05)**
$x^2 + y^2 + z^2 = a^2$ by the cone $x^2 + y^2 = z^2$.

b) Find by double integration the area inside the circle $r = a \sin\theta$ and outside **(05)**
the cardioid $r = a(1 - \cos\theta)$.

OR

Find the area bounded by the curve $x(x^2 + y^2) = a(y^2 - x^2)$ and its **(10)**
asymptote. Also find the area of the loop of the curve.

Q. 3 **a)** Prove that: $A \times (B \cap C) = (A \times B) \cap (A \times C)$. **(05)**

b) Using Venn diagram, prove or disprove : **(05)**
$A \oplus (B \oplus C) = (A \oplus B) \oplus C$.

OR

a) Prove that: $P \to (q \to r)$ and $(p \wedge \bar{r}) \to \bar{q}$ are logically equivalent. **(05)**

b) Construct the truth table for the following statement: **(05)**
$(\sim q \to \sim p) \to (p \to q)$.

Q. 4 **a)** Draw logic circuit diagram for the following expression **(05)**
$Y = ab + \bar{b}c + \bar{c}\,\bar{a}$.

P. T. O.

b) Minimize the following function using a K - map **(05)**

$F(W, X, Y, Z) = \sum(0, 4, 8, 12)$.

OR

Express $\bar{X}Y + Y\left(\bar{Z}\left(\bar{Z} + Y\right)\right)$ into canonical product – of – sums form. **(10)**

Q. 5 **a)** Show that : $a(-b) = (-a)b = -ab \ \forall \ a, b \in R$, where R is a ring. **(05)**

b) Consider $G = \{1, 5, 7, 11, 13, 17\}$ under multiplication modulo 18. **(05)**

Find $5^{-1}, 7^{-1}$.

OR

a) Let G_1 and G_2 be subgroups of a group G. Show that $G_1 \cap G_2$ is also a subgroup of G. **(05)**

b) Let G be a group with multiplication. Show that: $(ab)^{-1} = b^{-1}a^{-1} \ \forall \ a, b \in G$. **(05)**

Q. 6 **a)** Prove the following by mathematical induction: **(05)**

$$1^3 + 2^3 + 3^3 + - - - + n^3 = \left[\frac{n(n+1)}{2}\right]^2.$$

b) Solve : $a_r + a_{r-1} + a_{r-2} = r2^r$. **(05)**

OR

a) Prove the following by mathematical induction: **(05)**

$$1^2 + 2^2 + 3^2 + - - - + n^2 = \left[\frac{n(n+1)(2n+1)}{6}\right].$$

b) Determine the discrete numeric functions corresponding to the following generation function: **(05)**

$$A(Z) = \frac{3 - 5Z}{(1 - 2Z - 3Z^2)}.$$

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